Control of Omni-Directional Mobile Platform with Four Driving Wheels

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Abstract

A mobile manipulator consisting of a 6 d.o.f. manipulator and an omni-directional moving platform is developed. As an omni-directional mechanism, four special wheels with free rollers are used. 12 rollers are used for each wheel and they can rotate freely around the circumference of their wheel. The four wheels are controlled by DC servo motors respectively in order to position the moving platform on the ground. The generalized coordinate of the platform is three, that is \((x,y,\phi)\), on the other hand, the number of control input is four. Therefore the system has torque redundancy which can be used for improving the control performance of the moving platform. A criterion for exploiting the redundancy is proposed to avoid the slippage of wheels, and fundamental experiments are done to show the control characteristics of the developed mobile platform.

1. INTRODUCTION

Mobile manipulators consisting of a manipulator placed on a moving platform have wide moving range and can perform dexterous tasks. As a moving platform, omni-directional type mechanism is preferable because of its high mobility. There are several types of omni-directional mobile robots[1]-[3], but most of their mechanisms are complicated and some of them may be easy to slip on the ground due to its unevenness.

The figure 1 shows the developed mobile platform. The mechanism itself is very simple. Four special wheels with free rollers are used. 12 rollers are used for each wheel and they rotate freely around the circumference of their wheel, which enables the omni-directional motion of the platform. There are many similar omni-directional mobile robots with three wheels[4],[5]. However, most of them are not suitable for mobile platforms because working space of a manipulator mounted on such a platform with three wheels is very narrow. In the proposed platform, wide working space for a mounted manipulator is realized. By using four actuators to control three parameters that are the position of the platform, \(x,y\) and the orientation \(\phi\), robustness for slippages of wheels due to uneven ground can be realized.

In Section 2, the abstract of the mechanism is introduced. In Section 3, the control characteristics of the platform are shown, and two control algorithms are discussed. In Section 4, some fundamental experimental results are shown to estimate the effect of the slippage of wheels.

2. ABSTRACT OF MOBILE MANIPULATOR

Figure 2 shows the developed mobile manipulator.

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A commercial industrial robot MOVE MASTER RV-E3-ST made by Mitsubishi Electric Corporation is mounted on the octagon-shaped moving platform. The weight of the manipulator is about 30kg and the payload is 3kg. It is position controlled by displacement commands during one sampling time 50ms from a computer connected through RS232C.

The width of mobile platform is 500mm and its weight is about 15kg. The diameter of the wheel is 20cm. Each wheel axis of the platform is connected to a geared DC servo motor through a coupling joint. The motors are controlled by servo controllers (made by iXs Research Corporation) to which motor angle commands or torque commands for each motor are sent from a host PC. The sampling time of the servo controller is 1ms. The configuration of the control system is shown in Fig.3.

3. CONTROL ALGORITHMS FOR MOBILE PLATFORM

3.1 KINEMATICS OF MOBILE MANIPULATOR

The kinematic model of the mobile platform is shown in Fig.4. The platform moves on a horizontal plane which is the xy plane of the reference coordinate system. The platform coordinate system $\Sigma_p$ is set in the center of the octagon-shaped platform. The relationship between the small displacement of the platform and that of the wheels is expressed as Eq. (1),

$$\Delta \theta = J \Delta x,$$  \hspace{1cm} (1)

where

$$J = \begin{bmatrix}
  0 & \frac{1}{r} & \frac{L}{r} & \cos \phi & \sin \phi & 0 \\
  \frac{1}{r} & 0 & \frac{L}{r} & \cos \phi & \sin \phi & 0 \\
  0 & \frac{1}{r} & \frac{L}{r} & -\sin \phi & \cos \phi & 0 \\
 -\frac{1}{r} & 0 & \frac{L}{r} & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
  \frac{-\sin \phi}{r} & \cos \phi & \frac{L}{r} \\
  \frac{-\cos \phi}{r} & \sin \phi & \frac{L}{r} \\
  \frac{\sin \phi}{r} & \cos \phi & \frac{L}{r} \\
  \frac{\cos \phi}{r} & \sin \phi & \frac{L}{r}
\end{bmatrix},$$

$$\Delta \theta = [\Delta \theta_1 \ \Delta \theta_2 \ \Delta \theta_3 \ \Delta \theta_4]^T,$$

$$\Delta x = [\Delta x \ \Delta y \ \Delta \phi]^T.$$  

J is the Jacobian matrix of the platform. \(r\) is the radius of the wheel and \(L\) is the distance from the origin of $\Sigma_p$ to each wheel.

The relationship between small displacements of the end-effector of the manipulator $\Delta x_e \in \mathbb{R}^5$ and those of the manipulator joints $\Delta q \in \mathbb{R}^6$ are written as Eq.(2),

$$\Delta \theta = J \Delta x_e \hspace{1cm} (x_e, y_e, z_e, \theta_e, \phi_e, \psi_e)$$
\[ \Delta x_e = T J_m \Delta q + U \Delta x, \]  
where

\[
T = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix},
R = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix},
U = \begin{bmatrix} 1 & 0 & -p_x e \sin \phi & -p_y e \cos \phi \\ 0 & 1 & p_x e \cos \phi & -p_y e \sin \phi \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\]

\( J_m \) is the Jacobian matrix of the manipulator. The variables with the super-script ‘\( p \)’ are expressed with reference to the platform coordinate. In the equation (2), the small displacement of the platform is used instead of that of wheels. This is because the number of d.o.f. of the platform is three though there are four wheels. Therefore, \( \Delta x \) is more suitable for expressing the independent parameters of the platform than the wheel angles \( \Delta \theta \in R^4 \).

Control of the platform is only considered here. It is easy to install Eq.(1) into the control system because this algorithm can be realized with popular position controllers. If the desired position of the platform is \( x_d(t) \) and the present position is \( x(t) \), the commanded motor angles \( \Delta \theta(t) \) are obtained by Eq.(3),

\[ \Delta \theta(t) = DJ(x_d(t) - x(t)), \]  
where \( D = diag(g \quad g \quad g \quad g) \) and \( g \) is a reduction ratio of the motor gear. By sending \( \Delta \theta(t) \) to the servo controller, each motor is controlled by the torque commands calculated by Eq.(4).

\[ \tau_i(t) = k_p \Delta \theta_i + k_v \Delta \dot{\theta}_i + k_i \int \Delta \theta_i dt \quad (i = 1,2,3,4) \]  

The equation (4) is the position based control law. \( k_p, k_v \) and \( k_i \) are the feedback gains for each motor.

Four rotational wheel angles are uniquely determined by three parameters those are the desired position and orientation of the platform. Therefore, if some of the wheels have rotational errors, slippages occur inevitably. It is because the number of wheel is more than that of degrees of freedom of the platform. As a result, positioning accuracy of the platform becomes worse. Such characteristics are very critical for localization by odometry.

On the other hand, the relationship between the wheel torques and the driving forces of the platform is expressed as Eq.(5).

\[ f = J^T \tau \]  

In order to achieve the desired trajectory \( x_d(t) \), force commands expressed by Eq.(6) should be given to the platform.

\[ f_d = M \ddot{x}_d + k_v (\dot{x}_d(t) - \dot{x}(t)) + k_p (x_d(t) - x(t)) \]
where $M$ is the inertia matrix of the platform, $k_v$ is the velocity feedback gain matrix, and $k_p$ is the position feedback gain matrix. From Eq.(6), the torques $\tau$ realizing $f_d$ becomes as Eq.(7),

$$\tau = J^T f_d + (I - J^T J^T) a,$$  

(7)

where $J^T$ is the pseudo-inverse matrix of $J^T$ and $a$ is an arbitrary vector. The first term of the right side expresses the least norm solution and the second term means torque redundancy. The control performance of the platform can be improved by using this redundant term. If the Eq.(7) is used for control, slippage does not occur even if the wheels have torque errors. Moreover, by changing the torque distribution ratios to the four wheels by using the redundant term, bad effect of wheel slippages due to uneven ground on positioning accuracy can be decreased.

### 3.2 CONTROL LAW FOR EXPLOITING REDUNDANCY OF MOBILE PLATFORM

Let the whole weight of the mobile manipulator be $m$ and the reaction forces from the ground to each wheel be $f_i$. Then,

$$m = \sum_{i=1}^{4} f_i.$$  

(8)

On the other hand, the maximum driving force vector $f_{\text{wimax}}$ which is generated at the contact point between the ground and the i-th wheel in the tangential direction is as Eq.(9),

$$f_{\text{wimax}} = \mu_i f_i t_i,$$  

(9)

where $\mu_i$ denotes the maximum friction coefficient between the i-th wheel and the ground and $t_i$ does the unit vector in the tangential direction of the i–th wheel. If all the friction coefficients of the wheels are the same value, the maximum driving force of each wheel is proportional to the reaction force. Therefore, for avoiding the slippage, the desirable driving forces should be proportional to the reaction forces from the ground. In the proposed system, the number of redundancy is just one, therefore, the second term of the right side of Eq.(7) can be expressed as Eq.(10).

$$(I - J^T J^T) a = \alpha n$$  

(10)

‘$\alpha$’ is an arbitrary scalar and $n$ is an unit vector in the space spanned by the column vector of the matrix $I - J^T J^T$. By using Eq.(10), the Eq.(7) is rewritten as Eq.(11).

$$\tau' = J^T f_d + \alpha n$$  

(11)

Then, the criterion $H(\alpha)$ for exploiting the redundant term can be expressed as Eq.(12).

$$H(\alpha) = \left[ \begin{array}{cccc} f_1/m & f_2/m & f_3/m & f_4/m \end{array} \right] [\tau']$$  

(12)

By determining $\alpha$ so that the value of $H$ becomes maximum, the optimal motor torques are obtained. The optimal $\alpha$ can easily be obtained by solving a quadratic equation in $\alpha$. Let the optimal $\alpha$ be $\alpha_{\text{opt}}$, then, the optimal control law can be written as Eq.(13),
\( \tau_{opt} = J^T f_d + \alpha_{opt} n \). \hspace{1cm} (13)

Substituting Eq.(7) into Eq.(13), the optimal motor torques are obtained.

4. EXPERIMENT

Some fundamental experiments were done to check the effect of slippages when the position based control law by Eq.(3) and (4).

To measure the position of the mobile manipulator, a CCD camera fixed under the ceiling and three LEDs fixed on the mobile manipulator are used. The position and orientation of the platform is obtained by the positions of LEDs in the images from the CCD camera. By comparing the position and orientation by the CCD camera to those by odometry, the effect of slippages can be estimated. Two experiments were done. In the first experiment, the moving platform moves 40cm along the y axis in Fig.5 at a constant speed. In the second experiment, the moving platform did along the line \( x=y \) until \( x=y=40 \text{cm} \). In the first experiment, just two motors were used. In this case, servo errors of motors do not cause the slippages. On the contrary, in the second experiments, four wheels were used for moving platform. Therefore, mutual servo errors may cause the slippages of wheels, which brings bad effect to odometry.

Experimental results are shown in Fig.6 and Fig.7. In Fig.6, the thick line means the time trajectory of the platform measured by odometry, but the actual terminal position is about 37.7cm which is indicated by a thin line. In Fig.7, the thick expresses the result measured by odometry and the thin line does the result by CCD camera. The actual positioning error is about 8.8cm after the platform has moved 56.6cm which is much larger than the former result. It is supposed that the difference between two results is caused by the slippages of wheels. However, more experiments under many different conditions should be done to obtain certain effects.

5. CONCLUSION

A mobile manipulator consisting of a 6 d.o.f. manipulator and an omni-directional moving platform with four special wheels was developed. Since the system has torque redundancy, a way of exploiting the redundancy for avoiding slippage of the wheels was proposed. Fundamental experiments were performed.
done to estimate the effect of the slippage of wheels. More experiments under various conditions should be done to obtain certain effects. The implementation of the proposed method of exploiting redundancy into the developed system and the control of the whole system including the manipulator will be the next steps.

RÉFÉRENCES